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and re-invert into the original circles. The common tangent inverts into a circle through the point and tangent to the given circles. There are, in general, four solutions.

Next, let A, B, C be the centers, and a, b, c the radii, of three given circles. Suppose a is not greater than b or c. Describe circles with radii $b-a$ and $c-a$, and centers B, C, respectively. Let X be the center and x the radius of a circle through A and tangent externally to these two circles. Then a circle, center X and radius $x-a$ will be tangent to the three given circles. By obvious modifications we can, in general, get seven more circles touching the given circles.

We can get the numerical values of the radii and the positions of the centers by means of the linear relations connecting inverse figures.

It would be interesting to consider all the possible cases arising in making these constructions. For example, in the case of the escribed circles of a triangle the sides of the triangle are the limits of three pairs of circles and the nine-point circle is one of the remaining two circles.

CALCULUS.

245. Proposed by FRANCIS RUST, C. E., Allegheny, Pa.

Prove or disprove:

$$\int_0^\infty \frac{dx}{\sqrt{(x^2-1)(k^2x^2-1)}} = 2F^I(k) + \sqrt{(-1)}.F^I[\sqrt{(1-k^2)}],$$

Legendre's notation, $0<k<1$.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

$$L=\int_0^\infty \frac{dx}{\sqrt{[(x^2-1)(k^2x^2-1)]}} = \int_0^1 \frac{dx}{\sqrt{[(1-x^2)(1-k^2x^2)]}}$$

$$+\int_1^\infty \frac{dx}{\sqrt{[(x^2-1)(k^2x^2-1)]}} = F^I(k) + \int_1^\infty \frac{dx}{\sqrt{[(x^2-1)(k^2x^2-1)]}}.$$

Let $x=\dfrac{1}{ky}$. Then $\displaystyle\int_0^\infty \frac{dx}{\sqrt{[(x^2-1)(k^2x^2-1)]}}$

$$=\int_0^{1/k} \frac{dy}{\sqrt{[(1-y^2)(1-k^2y^2)]}} = F^I(k) + \int_1^{1/k} \frac{dy}{\sqrt{[(1-y^2)(1-k^2y^2)]}}.$$

$$\therefore L = 2F^I(k) + \int_1^{1/k} \frac{dy}{\sqrt{[(1-y^2)(1-k^2y^2)]}}.$$

Let $y=\dfrac{1}{\sqrt{(1-k'^2z^2)}}$, where $k'=\sqrt{(1-k^2)}$.

$$\therefore dy=\frac{k'^2 z dz}{(1-k'^2 z^2)^{\frac{3}{2}}},\quad \frac{1}{\sqrt{[(1-y^2)(1-k^2y^2)]}}=\frac{\sqrt{(-1)}(1-k'^2z^2)}{k'^2 z\sqrt{(1-z^2)}}.$$

$$\therefore \int_1^{1/k}\frac{dy}{\sqrt{[(1-y^2)(1-k^2y^2)]}}=\sqrt{(-1)}\int_0^1\frac{dz}{\sqrt{[(1-z^2)(1-k'^2z^2)]}}$$

$$=\sqrt{(-1)}F^I(k').$$

$$\therefore L=2F^I(k)+\sqrt{(-1)}F^I[\sqrt{(1-k^2)}].$$

Also solved by the Proposer.

246. Proposed by C. N. SCHMALL, 89 Columbia Street, New York City.

Derive Taylor's Series by the use of the formula for successive integration by parts, and nothing else.

Solution by the PROPOSER.

Assume the identity, $F(a+h)-F(a)=\int_a^{a+h}F'(x)dx$, which on integrating the right hand member by parts,

$$-\Big[(a+h-x)F'(x)\Big]_a^{a+h}+\int_a^{a+h}(a+h-x)F''(x)dx$$

$$=hF'(a)-\left[\frac{1}{2\,!}(a+h-x)^2F''(x)\right]_a^{a+h}+\frac{1}{2!}\int_a^{a+h}(a+h-x)^2F'''(x)dx$$

$$=hF'(a)+\frac{h^2}{2\,!}F''(a)+\ldots.+\frac{h^n}{n\,!}F^n(a)+\frac{1}{n!}\int_a^{a+h}(a+h-x)^nF^{n+1}(x)dx.$$

Hence, $F(a+h)-\left[F(a)+h\,F'(a)+\frac{h^2}{2\,!}F''(a)+\ldots+\frac{h^n}{n\,!}F^n(a)\right]$

$$(=R)=\frac{1}{n\,!}\int_a^{a+h}(a+h-x)^nF^{n+1}(x)dx.$$

We have assumed here that $F(x)$, $F'(x)$, $F''(x)$, ..., $F^n(x)$, $F^{n+1}(x)$, are all finite and continuous between the limits a and $(a+h)$ of x; now, if we put $A=F^{n+1}(a+\theta h)$, a mean value of $F^{n+1}(x)$ between the limits a and $(a+h)$, we have

$$R=\frac{1}{n!}\int_a^{a+h}(a+h-x)^nAdx=\frac{h^{n+1}}{(n+1)\,!}F^{n+1}(a+\theta h).$$

Also solved by Francis Rust.